

Asymptotically flat anisotropic space-time in 5 dimensions

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Abstract

We construct and investigate non conformal anisotropic Bianchi type VII solutions in 5 dimensions. The solutions are asymptotically flat, but they contain a naked singularity at the origin. We also construct solutions of Einstein-Maxwell gravity using the method employed in Majumdar - Papapetrou solutions with various profiles of charged dust. In a fictitious case of negative matter density, we obtain a solution with horizon hiding the singularity.

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1 Introduction

Understanding of nature using physics has revealed its richness and beauty in many ways. General relativity encodes the interaction of matter or radiation with space time in a charming manner. Though it is observed that many anisotropic solutions do exist in nature, spherically symmetric solutions have received more focus till now because of their simplicity. Homogenous, but anisotropic solutions of Einstein equations were classified by Bianchi many decades ago.[1, 2] However, finding such solutions have always been challenging and has led to some numerical results previously. Recently, anisotropic space times have gathered a renewed interest from a different direction. Many solutions of general relativity are much sought for the study of field theory using AdS/CFT correspondence as well as attractor mechanism.[3, 4, 5], Recently , asymptotic AdS (anti-de Sitter), anisotropic solutions were constructed and studied to investigate properties of certain anisotropic condensed matter systems using AdS/CFT correspondence. Anisotropic spacetimes are interesting systems and may contain many unique properties. Given the difficulty in their construction in 4 dimensions, and recent constructions of anisotropic solutions in 5 dimensions hint that that it may be less restrictive to construct anisotropic solutions in higher dimensions.

Asymptotically AdS, Bianchi *VII* class of solutions has received much attention as they were related to spatially modulated superconductors where the Cooper pair is not an s-wave, but a p-wave. The numerical solutions were constructed and their properties were further studied.[6, 7, 8, 9] This motivated us that similar anisotropic solutions with asymptotically flat or de Sitter property can also be constructed. We take up the investigation of such solutions in this manuscript.

In this manuscript, we report certain analytic Bianchi *VII*₀ solutions, but they contain a naked singularity. We further search for anisotropic solutions of Einstein Maxwell theory and construct few such solutions which can be considered as generalizations of Majumdar Papapetrou solutions for our case.[10, 11, 12, 13] We seek to construct solutions with a regular horizon hiding any naked singularity and we report one such case, though it is sourced by a fictitious matter density.

2 Bianchi Classes

Bianchi classified three dimensional homogeneous spaces into different classes. We present its brief review in this section. As we know that homogeneity means identical metric properties at all points of the space. Mathematically, homogeneity means the form of the metric does not change under translations. Translations along all directions are isometries and they are generated by the Killing vectors written in notations of differential geometry as

$$X_a = e_a^\alpha \frac{\partial}{\partial x^\alpha} \tag{1}$$

If we do not assume isotropy, then these Killing vectors will not commute in general. The commutators of generators can be represented as

$$[X_a, X_b] = C_{ab}^c X_c \quad (2)$$

where C_{ab}^c are structure constants. They are antisymmetric in lower indices i.e.

$$C_{ab}^c = -C_{ba}^c \quad (3)$$

and they also satisfy Jacobi identity,

$$C_{ab}^e C_{ec}^d + C_{bc}^e C_{ea}^d + C_{ca}^e C_{eb}^d = 0. \quad (4)$$

The structure constants are written in a form which separates its symmetric and anti-symmetric parts as follows,

$$C_{ab}^c = \epsilon_{abd} n^{dc} + \delta_b^c a_a - \delta_a^c b_b \quad (5)$$

where, ϵ_{abd} is the unit antisymmetric tensor, n^{ab} is a symmetric tensor with eigenvalues $n^{(1)}$, $n^{(2)}$ and $n^{(3)}$ and a_a is a vector. Next, a choice of frames is made with eigenvalues of n_{ab} chosen as the basis. The vector a_a is then chosen to be along a certain direction i.e. $\vec{a} = (a, 0, 0)$. The Jacobi identity gets reduced to $n^1 a = 0$. Thus, either a or n_1 has to vanish. The commutators of generators can then be explicitly written as

$$\begin{aligned} [X_1, X_2] &= aX_2 + n^{(3)}X_3, \\ [X_2, X_3] &= n^{(1)}X_1, \\ [X_3, X_1] &= n^{(2)}X_2 - aX_3. \end{aligned} \quad (6)$$

Using scale transformations and choice of signs, the list of possible types of homogeneous spaces turn out to be

type	a	n1	n2	n3
I	0	0	0	0
II	0	1	0	0
III	a	0	1	-1
IV	1	0	0	1
V	1	0	0	0
VI	a	0	1	-1
VII	a	0	1	1
VIII	0	1	1	-1
IX	0	1	1	1

We next choose to work with Bianchi type VII_0 , which chooses a and n^i both to be zero. We explicitly write the three vectors below.

$$\begin{aligned} X_1 &= \partial_1, \\ X_2 &= \cos(kx_1)\partial_2 + \sin(kx_1)\partial_3, \\ X_3 &= -\sin(kx_1)\partial_2 + \cos(kx_1)\partial_3. \end{aligned} \quad (7)$$

Henceforth, we will investigate solutions containing such anisotropy.

3 Anisotropic solution of pure gravity action

We consider a pure gravity action in five dimensions. It is known to be hard to construct solutions in 4 dimensions. But, recent progress have shown that the restrictions are little relaxed in higher dimensions. Our action is

$$S = \int \sqrt{|g|}(R) \quad (8)$$

Here, R is the Ricci scalar and g denotes determinant of the metric. The signature of our metric is $(-1,1,1,1,1)$. We choose our ansatz for the metric to be

$$ds^2 = -e^{2T(r)}dt^2 + dr^2 + e^{2M(r)+2N(r)}\omega_1^2 + e^{2M(r)-2N(r)}\omega_2^2 + e^{2Z(r)}\omega_3^2, \quad (9)$$

where, $T(r)$, $M(r)$, $N(r)$ and $Z(r)$ are chosen to be functions of r only. The one forms used above are

$$\begin{aligned} \omega_1 &= \cos(kx_1)dx_2 + \sin(kx_1)dx_3, \\ \omega_2 &= -\sin(kx_1)dx_2 + \cos(kx_1)dx_3, \\ \omega_3 &= dx_1. \end{aligned} \quad (10)$$

Thus, we are looking for static, anisotropic Bianchi VII_0 solutions since we are restricting the appearance of coordinates x_2 and x_3 in the metric in the above specific combinations only. We choose to work in a non-coordinate basis with the following vielbeins. The metric in such a basis is diagonal. Our vielbeins are

$$\begin{aligned} e^t &= e^{T(r)}dt, \\ e^r &= dr, \\ e^a &= e^{M(r)+N(r)}\omega_1, \\ e^b &= e^{M(r)-N(r)}\omega_2, \\ e^c &= e^{Z(r)}\omega_3. \end{aligned}$$

We get the following Einstein equations of motion

$$\begin{aligned} T''G' &= 0, \\ M''G' &= 0, \\ N''G' - k^2e^{-2Z}\sinh 4N &= 0, \\ Z''G' + 2k^2e^{-2Z}\sinh^2(2N) &= 0, \\ G''^2 + 2k^2e^{-2Z}\sinh^2(2N) &= 0, \end{aligned} \quad (11)$$

including a constraint

$$G'^2 + (T'^2 + 2M'^2 + 2N'^2 + Z'^2) + \frac{k^2}{2}(e^{(2N-Z)} - e^{(-2N-Z)})^2 = 0. \quad (12)$$

Here, variable G denotes $T + 2M + Z$ and superscript prime denotes derivative with respect to r . The first two equations suggest that a good radial variable will be u defined as

$$du = e^{-G} dr. \quad (13)$$

Also, we define a different variable $L = G - Z$. Then, the set of equations are

$$\begin{aligned} T_{uu} &= 0, \\ M_{uu} &= 0, \\ L_{uu} &= 0, \\ N_{uu} - k^2 e^{2L} \sinh 4N &= 0, \\ Z_{uu} + 2k^2 e^{2L} \sinh^2 2N &= 0, \\ G_{uu} + 2k^2 e^{2L} \sinh^2 2N &= 0. \end{aligned} \quad (14)$$

The constraint equation now becomes

$$G_u^2 + T_u^2 + 2M_u^2 + 2N_u^2 + Z_u^2 + 2k^2 e^{2L} \sinh^2(2N) = 0. \quad (15)$$

We then proceed to express the functions T , M and L as

$$\begin{aligned} T &= t_0 + 2t_1 u, \\ M &= m_0 + m_1 u, \\ L &= a + l_1 u, \end{aligned} \quad (16)$$

where t_0 , t_1 , m_0 , m_1 , a and l_1 are constants. We find the next amenable equation to solve is that for N . However, this equation become simpler if one chooses the parameter l_1 to be 0 or function $L(r)$ to be a constant function. The equation for N then reduces to

$$N_{uu} - \lambda^2 \sinh(4N) = 0, \quad (17)$$

where $\lambda = ke^a$ is a constant. Inverting this differential equation results in

$$\frac{u_{NN}}{u_N^3} + \lambda^2 \sinh(4N) = 0, \quad (18)$$

which if integrated once, leads to

$$\lambda^2 u_N^2 = \frac{2}{(c_1 + \cosh(4N))}. \quad (19)$$

Here, c_1 is an integration constant. The above equation can be solved in terms of Jacobi amplitudes. However, it offers simple solution for two cases, (1) $c_1 = -1$ and (2) $c_1 = 1$. We next proceed to explore the case of $c_1 = -1$ in more detail. We can then write the above equation as

$$\lambda u_N = \frac{1}{\sinh(2N)} \quad \text{or} \quad N_u = \lambda \sinh(2N). \quad (20)$$

Its general solution is

$$e^{-2N} = \tanh(\lambda u + u_0). \quad (21)$$

It can also be written as

$$2 \sinh[2(\lambda u + u_0)] \sinh(2N) + 1 = 0. \quad (22)$$

The equation for Z can be written as

$$Z_{uu} = -\frac{2\lambda^2}{\sinh^2[2(\lambda u + u_0)]}. \quad (23)$$

It has the general solution

$$Z = Z_0 + Z_1 u + \frac{1}{2} \ln \sinh[2(\lambda u + u_0)]. \quad (24)$$

Since L was taken as a constant and $G = L + Z$, variable G also gets determined. Most of the free parameters get fixed by the constraint relation, which now takes the form

$$G_u^2 - (T_u^2 + 2M_u^2 + 2N_u^2 + Z_u^2) + \frac{k^2}{2} e^{2a} \sinh^2(2N) = 0. \quad (25)$$

Since, $G_u = Z_u = T_u + 2M_u + Z_u$, we get $t_1 = -2m_1$. One can deduce using eqn. (20) that $m_1^2 = 0$. Further, we can absorb the parameters Z_0 , t_0 and Z_1 by redefining coordinates x_3 , t and u respectively. Furthermore, variable m_0 can be absorbed by redefining coordinates x_2 and x_3 . Moreover, redefining the constant a , the metric can be written in a form

$$ds^2 = -a^2 dt^2 + e^{2u} \sinh(2\lambda u)(a^2 du^2 + \omega_3^2) + \frac{1}{\tanh(\lambda u)} \omega_1^2 + \tanh(\lambda u) \omega_2^2. \quad (26)$$

This metric is Ricci flat ($R_{\mu\nu} = 0$) but contains a naked singularity at the origin. The Kretschmann tensor blows at $u = 0$. However, we notice that the metric component along the time direction is just a constant. In fact, we have a spatial 4 dimensional submanifold with a naked singularity in addition with a time direction. We hope that the naked singularity can still be put in a physical context if we excite some other field whose energy density itself becomes infinity at $u = 0$ thus causing strong curvature there. If it happens, there is hope that this singularity can be hidden behind a horizon.

4 Anisotropic solution of Einstein Maxwell action

We will now try to construct anisotropic solutions of 5 dimensional Einstein Maxwell action along with matter density. Our action in this section is

$$S = \int d^5x \sqrt{-g} \left[R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu - \frac{\rho}{2} (g^{\mu\nu} u_\mu u_\nu + 1) \right]. \quad (27)$$

Here, notation g denotes the determinant of the metric which we choose to have one negative signature along time direction as earlier. Ricci scalar is denoted by R . The

electromagnetic potential and field strength are denoted by A_μ and $F_{\mu\nu}$, respectively. We also incorporate source for electromagnetic field denoted as J_μ as well as a matter density denoted by ρ with a velocity u_μ . We take the velocity to be non dynamical field. The matter term in action is conspired to give the correct energy momentum tensor for a pressureless dust i.e. $T_{\mu\nu} = \rho u_\mu u_\nu$. The Maxwell equation is

$$\nabla_\nu F^{\mu\nu} = J^\mu. \quad (28)$$

The metric fluctuations of the action leads to following Einstein equations.

$$R_{\mu\nu} = \frac{1}{2}F_{\mu\rho}F_\nu{}^\rho - \frac{1}{12}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + \rho u_\mu u_\nu + \frac{1}{3}g_{\mu\nu}\rho. \quad (29)$$

We further attempt to find solutions of these set of equations using a method employed earlier to find generalizations of Majumdar-Papapetrou metrics.[12, 13, 14] We mention that Majumdar Papapetrou metrics are 4 dimensional extremal solutions of Einstein-Maxwell equations of the type

$$ds^2 = -V(\vec{x})^2 dt^2 + \frac{1}{V(\vec{x})^2}(dx_1^2 + dx_2^2 + dx_3^2) \quad (30)$$

alongwith an electromagnetic flux of the kind $F = d(V^{-1})$. The function V is required by Einstein equations to be a harmonic function of the 3 dimensional flat subspace. When searching for solutions in $(m+1)$ dimensions, we generalize the metric ansatz to be of type

$$ds^2 = -V(x_i)^2 dt^2 + \frac{1}{V^{2/n}(x_i)} h^{ij} dx_i dx_j \quad i = 1, 2, \dots, m. \quad (31)$$

The metric h_{ij} depends on spatial coordinates only. It leads to the following Ricci tensor,

$$\begin{aligned} R_{tt} &= V^{(1+2/n)} \nabla_{(h)}^2 V - \frac{m-2}{n} V^{2/n} (\nabla V)^2 \\ R_{ij} &= R_{ij}^{(h)} - \frac{(mn+2-m)}{n^2 V^2} \partial_i V \partial_j V + h_{ij} \left[\frac{\nabla_{(h)}^2}{nV} - \frac{(m-2)}{n^2} \left(\frac{\nabla V}{V} \right)^2 \right] \\ &\quad + \frac{(m-2-n)}{nV} \nabla_i \nabla_j V. \end{aligned} \quad (32)$$

The indices i, j denote spatial coordinates only. Here, notation $\nabla_{(h)}^2$ denotes Laplacian defined over the internal space with metric h_{ij} . The Ricci tensor component R_{ti} is found to be vanishing. When we write the corresponding energy momentum tensor $T_{\mu\nu}$ and try to satisfy Einstein equations, we notice that there is no analog of any term like $\nabla_i \nabla_j V$ in the expression of $T_{\mu\nu}$. Such a term, if kept, will require us to solve complicated non-linear equations. One chooses a relation between parameters m and n , so as to make such term vanish i.e. $n = m - 2$. Returning to our interest of 5 dimensional metrics, we find that we should take $m = 4$ and $n = 2$. Thus, our metric ansatz reduces to

$$ds^2 = -V(x_i)^2 dt^2 + \frac{1}{V(x_i)} h^{ij} dx_i dx_j \quad (33)$$

We next evaluate the components of the Einstein tensor and they are found to be

$$\begin{aligned} G_{00} &= \frac{3}{2}V^2\nabla_h^2V - \frac{9}{4}Vh^{ij}\partial_iV\partial_jV + \frac{1}{2}V^3R_{(h)} \\ G_{ij} &= R_{(h)ij} - \frac{3}{2V^2}\partial_iV\partial_jV + \frac{3}{4V^2}h_{ij}h^{kl}\partial_kV\partial_lV - \frac{1}{2}h_{ij}R_{(h)}. \end{aligned} \quad (34)$$

We choose our internal subspace to be the anisotropic Ricci flat space that we obtained in last section. Thus, Ricci tensor $R_{(h)}$ and Ricci scalar $R_{(h)}$ vanishes. The Einstein tensor in our case then reduces to

$$\begin{aligned} G_{00} &= \frac{3}{2}V^2\nabla_h^2V - \frac{9}{4}Vh^{ij}\partial_iV\partial_jV \\ G_{ij} &= -\frac{3}{2V^2}\partial_iV\partial_jV + \frac{3}{4V^2}h_{ij}h^{kl}\partial_kV\partial_lV \end{aligned} \quad (35)$$

Next we make an ansatz for the electromagnetic potential A_μ . We assume it to be along the time direction

$$A_\mu = A\delta_\mu^0. \quad (36)$$

We also make an ansatz for the four velocity of the matter density. We assume matter to be at rest i.e.

$$u_\mu = V\delta_\mu^0 \quad (37)$$

Such a choice also ensures that $u_\mu u^\mu = -1$. The energy momentum tensor component from electromagnetic field is given in terms of field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as

$$T_{\mu\nu}^{field} = \frac{1}{4\pi}(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}). \quad (38)$$

The matter energy density contribution to energy momentum tensor is

$$T_{\mu\nu}^{matter} = \rho u_\mu u_\nu. \quad (39)$$

Thus the components of total energy-momentum tensor turn out to be

$$\begin{aligned} T_{00} &= \frac{1}{3}Vh^{ij}\partial_iV\partial_jV + \frac{2}{3}\rho V^2 \\ T_{ij} &= -\frac{1}{2V^2}\partial_iV\partial_jV + \frac{h_{ij}}{6V^2}\{(\nabla_h A)^2 + 2\rho V\} \end{aligned} \quad (40)$$

The non trivial components of Einstein equations are

$$V^2\nabla_h^2V - V(\nabla_h V)^2 = \frac{1}{3}V(\nabla_h A)^2 + \frac{2}{3}\rho V^2 \quad (41)$$

$$\frac{3\partial_iV\partial_jV}{2V^2} - \frac{h_{ij}}{2V^2}\{V\nabla_h^2V - (\nabla V)^2\} = \frac{\partial_iA\partial_jA}{2V^2} - \frac{h_{ij}}{6V^2}\{(\nabla_h A)^2 + 2\rho V\} \quad (42)$$

We notice that the term explicitly proportional to h_{ij} in equation (42) is same as (tt) component Einstein equation as in (41). Canceling it, we get

$$\frac{3\partial_iV\partial_jV}{2V^2} = \frac{\partial_iA\partial_jA}{2V^2}. \quad (43)$$

It can be solved easily if we take A proportional to V . The equation fixes the relation to be $A = \sqrt{3}V$. Then the rest of Einstein equations simplifies to

$$\nabla_h^2 \left(\frac{1}{V} \right) = -\frac{2\rho}{3V^2}. \quad (44)$$

In terms of $\lambda = \frac{1}{V}$, the above equation can be written as

$$\nabla_h^2 \lambda = -\frac{2\rho\lambda^2}{3}. \quad (45)$$

We next make an ansatz for the source of the electromagnetic field. We take only the time component of J_μ to be non trivial. Alongwith the above choice for electromagnetic potential, the Maxwell equation takes a form

$$\nabla_h^2 \lambda = -\frac{\lambda J^t}{\sqrt{3}}. \quad (46)$$

This equation will be consistent with the equation (45), if we chose

$$J^t = \frac{2\rho\lambda}{\sqrt{3}} \quad (47)$$

Thus we are left with a single equation viz. equation (45), which is a non homogenous Laplacian equation. We next proceed to solve it for some suitable choices of matter density.

5 Anisotropic solutions with chosen sources

5.1 Polynomial solutions

The equation we need to solve is

$$\nabla_h^2 \lambda = \frac{1}{\sqrt{h}} \partial_i (\sqrt{h} h^{ij} \partial_j \lambda) = -\frac{2\rho\lambda^2}{3}. \quad (48)$$

We choose our spatial subspace to be same as the anisotropic 4 dimensional subspace which was obtained in the second section. Therefore,

$$h_{ij} dx^i dx^j = e^{2u} \sinh(2\lambda u) (a^2 du^2 + dx_1^2) + \frac{1}{\tanh(\lambda u)} \omega_1^2 + \tanh(\lambda u) \omega_2^2. \quad (49)$$

The determinant of the metric is $ae^{2u} \sinh(2\lambda u)$. We will henceforth denote x_1 simply by x . For simplicity, we assume the function λ to be a function of u and x only.

Then equation (48) becomes,

$$\frac{1}{\sqrt{h}} \left(\frac{\partial_u^2 \lambda}{a} + a \partial_x^2 \lambda \right) + \frac{2}{3} \rho \lambda^2 = 0 \quad (50)$$

We next define polar coordinates $au = r \cos \phi$ and $x = r \sin \phi$. The equation appears in polar form as

$$\partial_r^2 \lambda + \frac{1}{r} \partial_r \lambda = -\frac{2}{3a} \sqrt{h} \rho \lambda^2 \quad (51)$$

where we have taken λ to be independent of ϕ , i.e. we restrict ourselves to the lowest harmonic. Now the equation can be made amenable to analytical results for suitably choosing the profiles for the matter density. We next choose

$$\rho = \frac{3ac}{2r^n \sqrt{h} \lambda^2} \quad (52)$$

where, n is a positive integer greater than 2 and c is a constant. Such a form of energy density is physically reasonable as it vanishes smoothly to zero when one proceeds towards infinity. Then equation (51) becomes,

$$\partial_r^2 \lambda + \frac{1}{r} \partial_r \lambda = -\frac{c}{r^n} \quad (53)$$

One can easily solve it to obtain

$$\lambda = -\frac{c}{(n-2)^2 r^{n-2}} + c_2 + c_1 \log r. \quad (54)$$

We can restrict ourselves to polynomial form by choosing $c_1 = 0$. This leads to

$$\lambda = c_2 - \frac{c}{(n-2)^2 r^{n-2}}. \quad (55)$$

By choosing $n = 3$, we get

$$V = \frac{1}{c_2 - \frac{c}{r}} \quad (56)$$

Then, the metric now appears as

$$ds^2 = -\frac{1}{(c_2 - \frac{c}{r})^2} dt^2 + (c_2 - \frac{c}{r}) h^{ij} dx_i dx_j. \quad (57)$$

But, this solution shows two essential singularity where Kretschmann tensor diverges. They are $r = 0$ and $r = c/c_2$. Thus there are two naked singularities in this solution. We next consider a fictitious matter whose density profile is negative by replacing the constant c with $-C$. This sends the second singularity at $r = c/(c_2)$ to a negative value of r , thus out of the spacetime. Choosing constant $c_2 = 1$, the metric now appears as

$$ds^2 = -\frac{1}{(1 + \frac{C}{r})^2} dt^2 + (1 + \frac{C}{r}) h^{ij} dx_i dx_j. \quad (58)$$

We find that the (tt) component of the metric for small values of r is

$$g_{tt} = \frac{1}{(1 + \frac{C}{r})^2} \sim 1 - \frac{2C}{r}. \quad (59)$$

Thus, this solution has a horizon near $r \sim 2C$, which also hides the essential singularity residing at $u = 0$. We expect this solution to be of extremal type as the same is true for all such previous generalizations of Majumdar Papapetrou metrics.

5.2 The Sine-Gordon Solution

One can get a Sine Gordon kind of equation here by a different choice of matter density. First we choose a different radial coordinate

$$\tau = \log r$$

Then the equation (51) becomes,

$$\partial_\tau^2 \lambda = -\frac{2}{3} e^{2\tau} \sqrt{h} \rho \lambda^2 \quad (60)$$

Next, we choose matter density profile to be of form

$$\rho = \frac{3a\delta^2 \sin \lambda}{2e^{2\tau} \sqrt{h} \lambda^2} \quad (61)$$

The above equation then reduces to a Sine-Gordon equation,

$$\partial_\tau^2 \lambda + \delta^2 \sin \lambda = 0. \quad (62)$$

It admits a solution

$$\lambda(\tau) = 4 \arctan \left[\tanh \left(\frac{\delta\tau + c}{2} \right) \right] \quad (63)$$

This ensures that g_{tt} is finite everywhere except at origin $r = 0$, where one encounters a naked singularity.

6 Conclusion

We have constructed an anisotropic 4 dimensional asymptotically flat Riemannian metric which is Ricci flat. We later incorporated it in a 5 dimensional space time along with matter density and electro magnetic flux using a method similar to Majumdar Papapetrou way of constructing extremal solutions. With certain choices of matter density profiles, we were able to construct explicit solutions. Most of them contain the naked singularity. However, with a fictitious choice of matter density, we arrived at a solution with a horizon hiding the naked singularity. Given a Laplacian equation with a negative source, we are more likely to come across naked singularity. For a horizon to appear, we generally need g_{tt} to vanish at some finite radial coordinate. This requires function V to vanish and its inverse, λ to blow up at some r . However, by a suitable choice of variables, we can write the Laplacian as a second derivative of λ . For a second derivative to be negative at all positions as dictated by its negative source, the function λ becomes a convex function. A function blowing up at a certain finite value generally should be concave instead of convex. Thus, when we take the source to be positive by taking the matter density negative, we arrive at a spacetime with horizon. Using our method, interesting anisotropic solutions in higher dimensions may be constructed by further investigating further types of Lagrangians which can hide the naked singularity present in the center of the subspace behind a horizon.

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